Simulation opto-mechanical cavity with an optical spring mirror

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Directed Research:

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Agenda

- Background/Project overview
- Modeling the cavity
- Variations on the model
- Results from preliminary calculations
- Conclusion
- Questions

Background/Project overview

Optical tweezers

- Optical tweezers are noninvasive tools that use a laser beams to generate Pico-Newton forces powerful enough to manipulate microscopic matter.
- It is possible to measure the elastic properties of DNA by grabbing hold of beads attached to the ends of the molecules and stretching them.

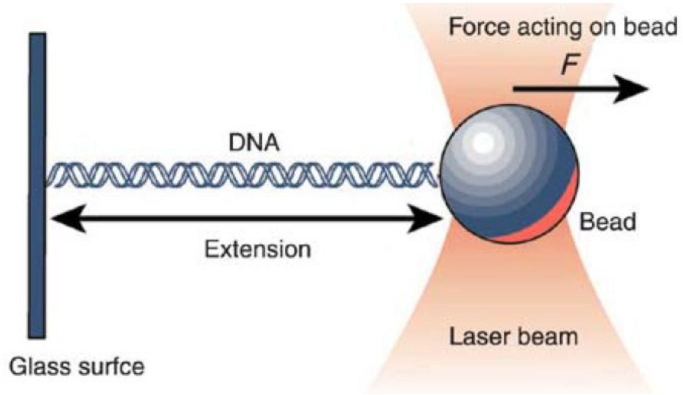
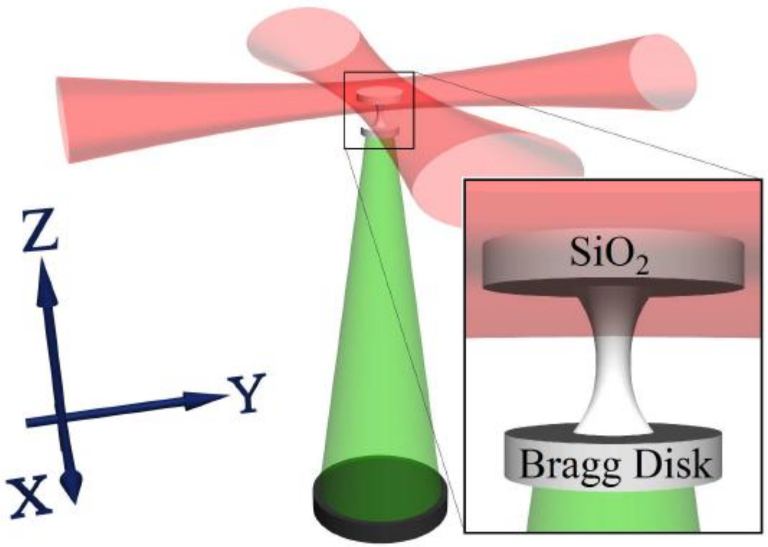


Illustration by Terry Miura

The project

- With the use of optical tweezers, it may be possible to suspend a mirror in a cavity.



Reasoning behind project

- Currently, a primary hurtle in the operation of opto-mechanical systems in the quantum regime is the coupling of the vibrating element to a thermal reservoir via mechanical supports.

Modeling the cavity

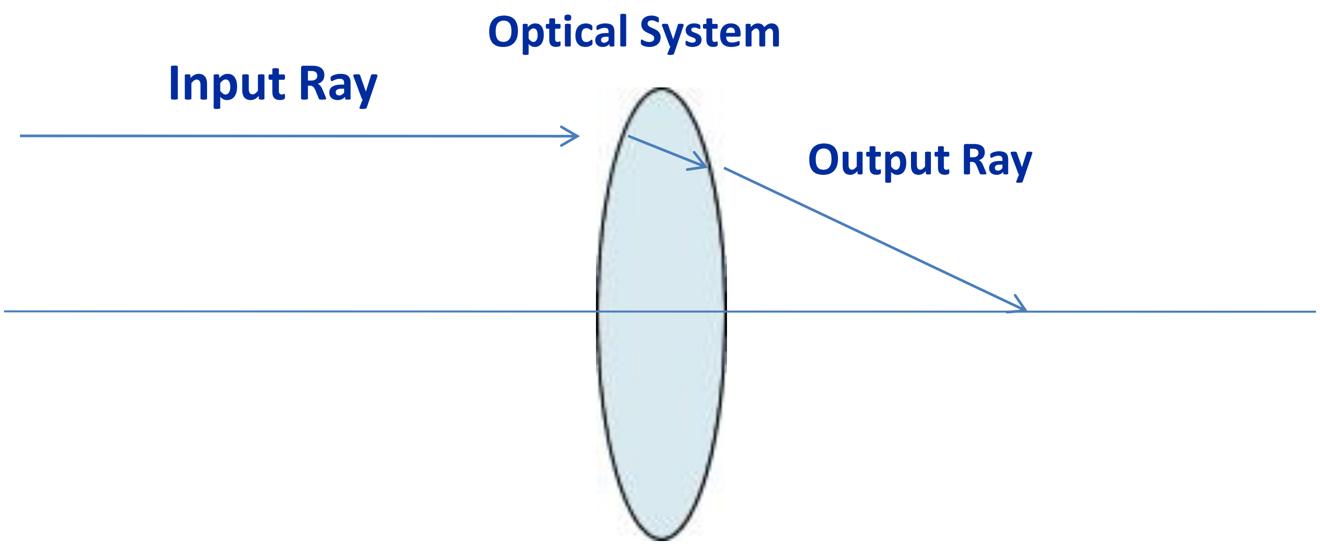
Modeling the Cavity

- Modeling the cavity was a three step process.
  – Describing cavity using ABCD matrix.
  – Conventional analysis of laser cavity.
    - ABCD law for Gaussian beams.
  – Generalization of Huygen's integral.
    - Allows us to account for phase variations in cavity to model implications of tilting mirror.

Ray transfer matrix analysis

- Ray transfer matrix analysis (also known as ABCD matrix analysis) is a type of ray tracing.
- It involves the construction of a ray transfer matrix which describes the optical system; a ray can then be traced through the system by multiplying this matrix with a vector representing the light ray.

ABCD matrix method

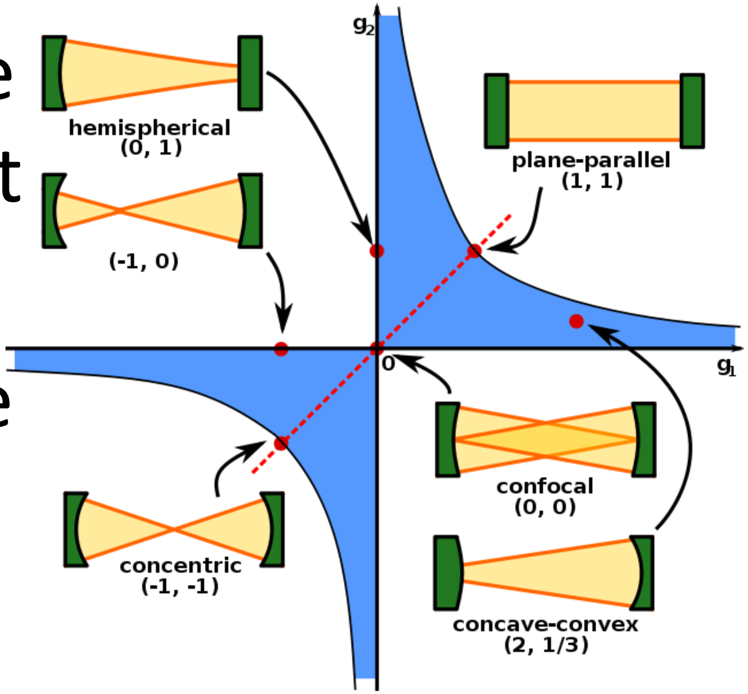


Input Ray — Optical System — Output Ray

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix}$$

Types of cavities

- There are many different types of cavities used in the construction of Fabry–Pérot resonators.
- Usually, cavity decisions are made based on cost and alignment considerations.



Cavity Decision

- Due to the nature of optical tweezers, the natural choice of cavity is hemispherical.
  - This decision allows for the smallest secondary mirror.

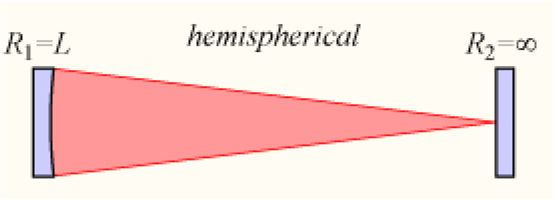
ABCD cavity model

- Modeling a periodic system such as a Fabry–Pérot cavity can be done using the ABCD model.
- All that is necessary is to model each mirror, then iterate the field inside using this model.

Element	Matrix
Propagation in free space or in a medium of constant refractive index	$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$
Refraction at a flat interface	$\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$
Refraction at a curved interface	$\begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R \cdot n_2} & \frac{n_1}{n_2} \end{pmatrix}$
Reflection from a flat mirror	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Reflection from a curved mirror	$\begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}$
Thin lens	$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$
Single right angle prism	$\begin{pmatrix} k & \frac{d}{nk} \\ 0 & \frac{1}{k} \end{pmatrix}$

ABCD cavity model

- With the decision to use a hemispherical cavity, we can derive the following two matrices depending on whether we want to know information at the flat mirror or the curved mirror.



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Bragg\ Mirror} = \begin{bmatrix} 1-\frac{2L}{R_1} & L+L\left(1-\frac{2L}{R_1}\right) \\ -\frac{2}{R_1} & 1-\frac{2L}{R_1} \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{Curved\ mirror} = \begin{bmatrix} 1 & 2L \\ -\frac{2}{R_1} & 1-\frac{4L}{R_1} \end{bmatrix}$$

ABCD law for Gaussian beams

- The ABCD law for Gaussian beams is a method for determining the stability and amplification of a laser resonator.
- Through Eigenvalue matrix analysis we can make the following statements about the
  - Find lambda to keep the input and output vectors constant.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

ABCD law for Gaussian beams

- The result of Eigenvalue analysis gives us the following stability conditions.

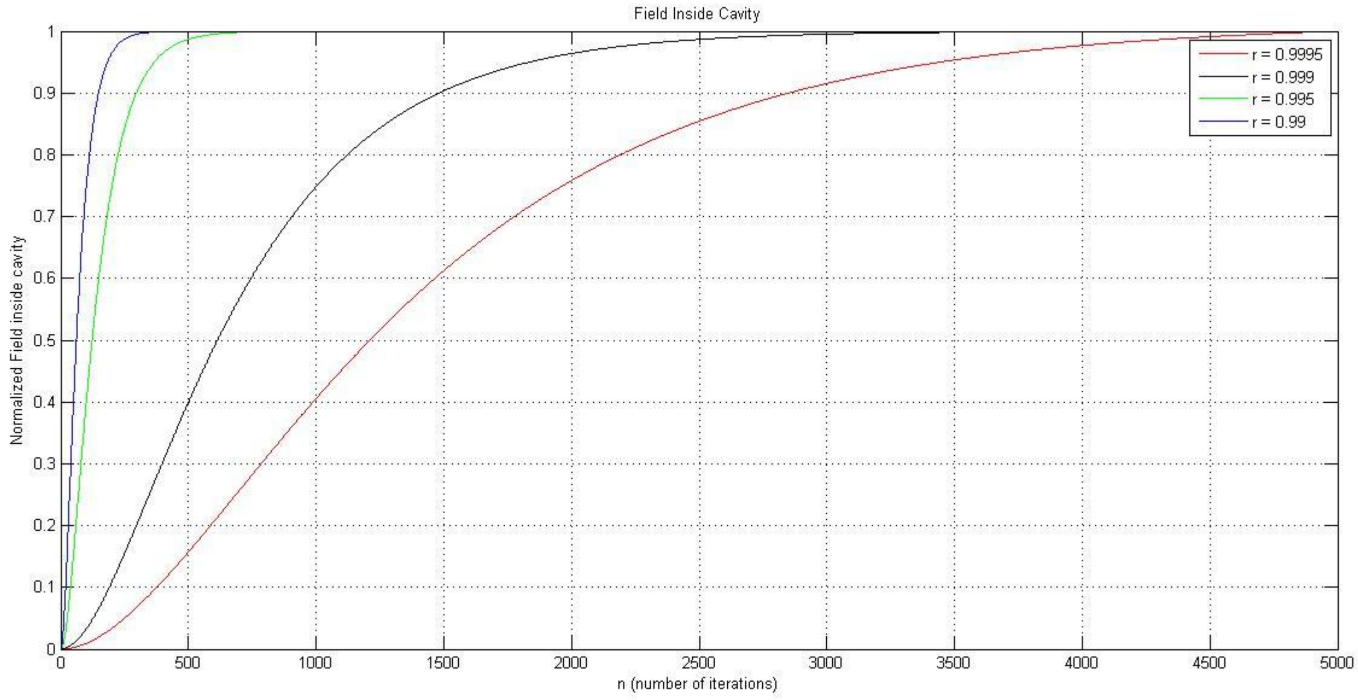
$$-1 \leq 1 - 2L/R \leq 1$$

- We can also determine from further analysis information about the spot size.

$$\omega_{spotsize} = \sqrt{\frac{k}{2|B|} * \sqrt{1 - \left(\frac{A+B}{2}\right)^2}}^{\,-1}$$

Results from first order analysis

- Computing power.



Results from first order analysis

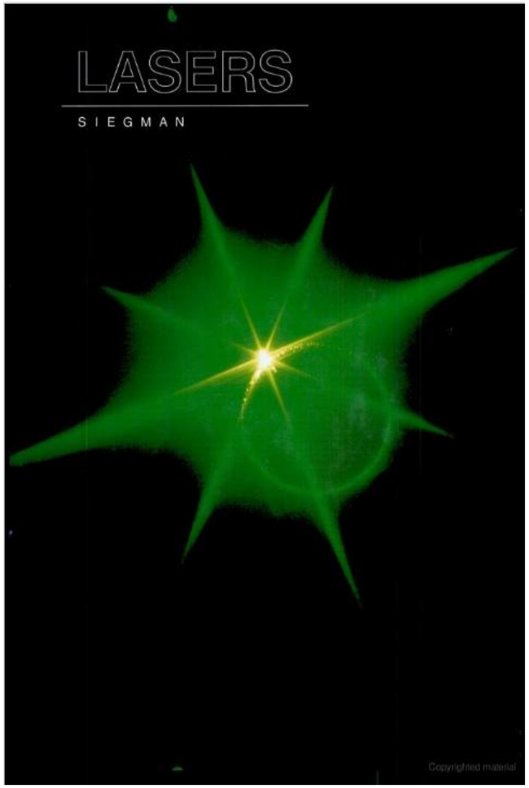
- Amplification of field depends on reflectivity of the mirrors.

Reflectance coefficients	Amplification
r = 0.9995	1000%
r = 0.999	500%
r = 0.995	100%
r = 0.99	50%

Generalization of Huygen's integral

- Allows us to make a more detailed analysis of the cavity using ABCD matrix.

$$E(x_1, z_1) = \sqrt{\frac{1}{i\lambda B}} \int_{-\infty}^{\infty} dx_0 e^{\frac{ik_0}{2B}[Ax_0^2 - 2x_0x_1 + Dx_1^2]} E(x_0, z_0)$$



Computing algorithm

- Numerical approach to solving the integral.
  - From the initial field we calculate

$$E'(x_0', y_0') = e^{\frac{ik}{2B}(A-1)x_0^2} E(x_0, z_0)$$

  - Propagate the field a distance B

$$\mathbb{E}(\xi, \eta) = \mathcal{F}\left(E'(x_0', y_0')\right) * e^{-\frac{iB\xi^2}{2k}}$$

Computing algorithm

- Multiply by a phase factor to get the output field.

$$E(x_1, y_1) = e^{\frac{ik}{2B}(D-1)x_1^2} * \mathcal{F}^{-1}\left(\mathbb{E}(\xi, \eta)\right)$$

- To get the field inside the Fabry-Pérot cavity, we multiply by the reflectance of each mirror and the phase acquired by one complete round trip.

$$E(x_1, y_1)_{round\ trip} = r_1 r_2 E(x_1, y_1) e^{i\phi}$$

Computing algorithm

- Finally, add the initial field to the round trip field and continue to iterate the process until the field inside the cavity equalizes.

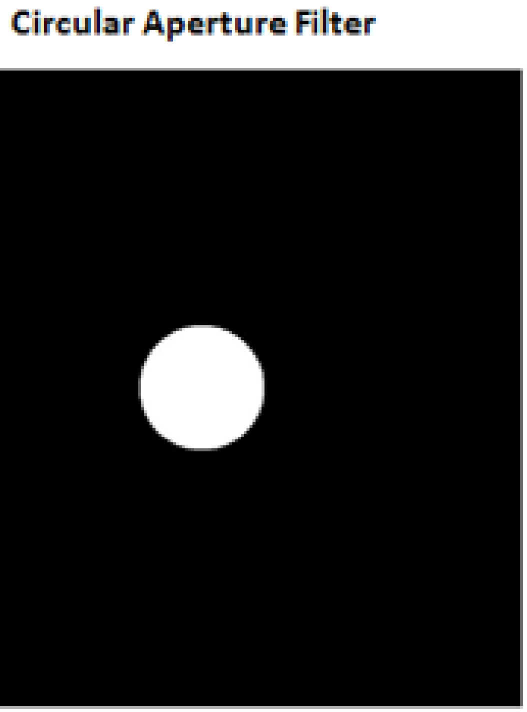
$$E(x,y)_{field} = E(x_1,y_1)_{round\ trip} + E(x_0,z_0)$$

Variations on model

Modeling mirror diameter

- To model the mirror size, we create a filter to only pass a portion of the Gaussian beam, similar to how a mirror would reflect only a portion of the beam.
- A matrix created correctly can accomplish this by using the number 1 to pass the modeled beam and the number 0 to kill it.

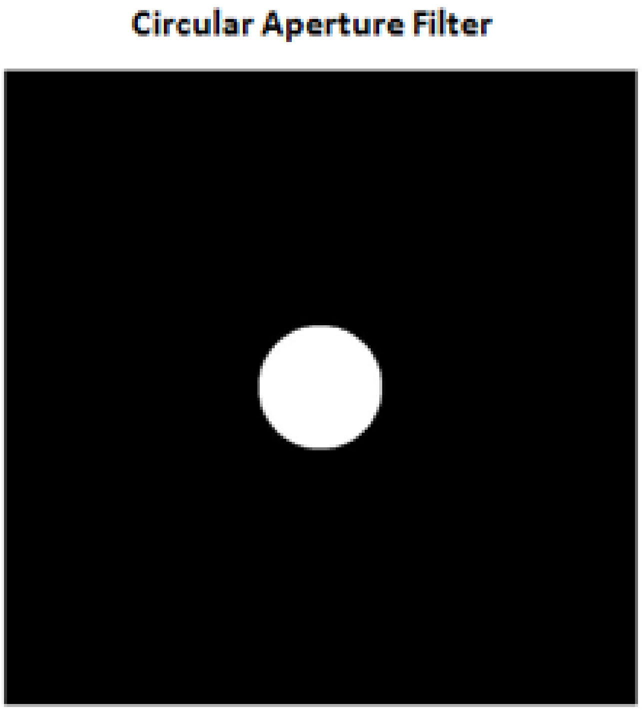
Modeling mirror diameter

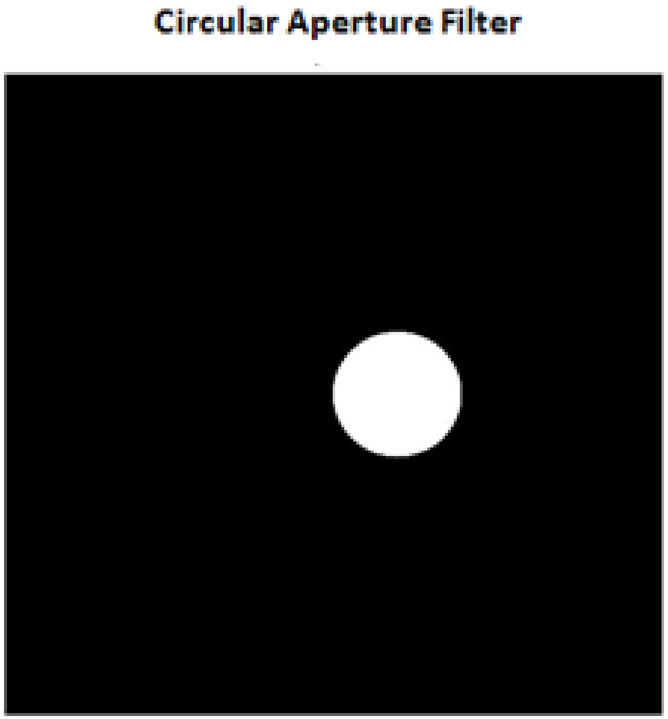


Modeling mirror shift

- To model the mirror aperture moving laterally, we can apply the same logic as we did for modeling the mirror size.
- We create a matrix that allows some of the beam to pass and some of the beam to be blocked. Then we simply translate the ones in the matrix so that the filter moves laterally similar to a mirror moving laterally.

Modeling mirror shift





Modeling mirror tilt

- Tilt causes an extra phase factor which can quickly degrade the efficiency of the cavity by causing destructive interference.
- Simply adding phase to the beam as a function of x can simulate mirror tilt.

$$E_{tilt} = E_0 * e^{(iksin(\theta)x)}$$

Results from preliminary calculations

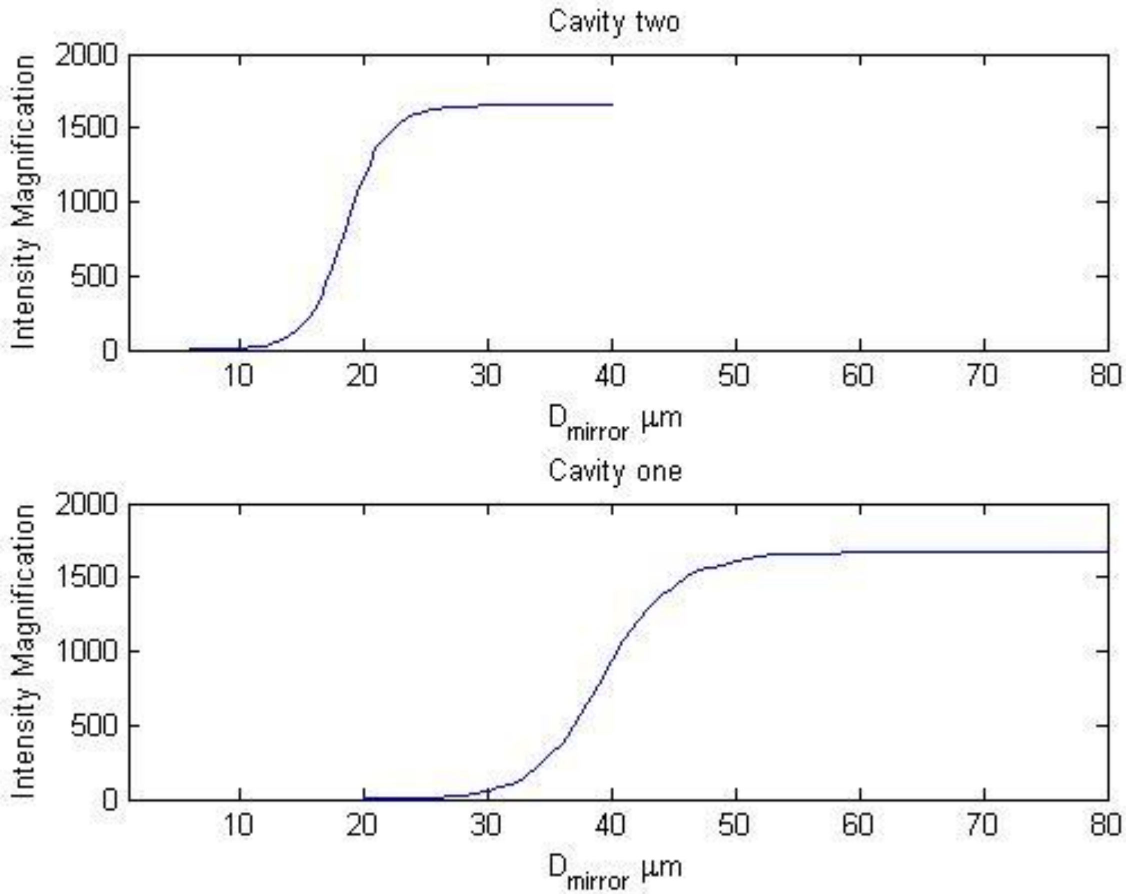
Specification on cavities modeled

- To run simulations, these cavities were chosen based on stability calculations.

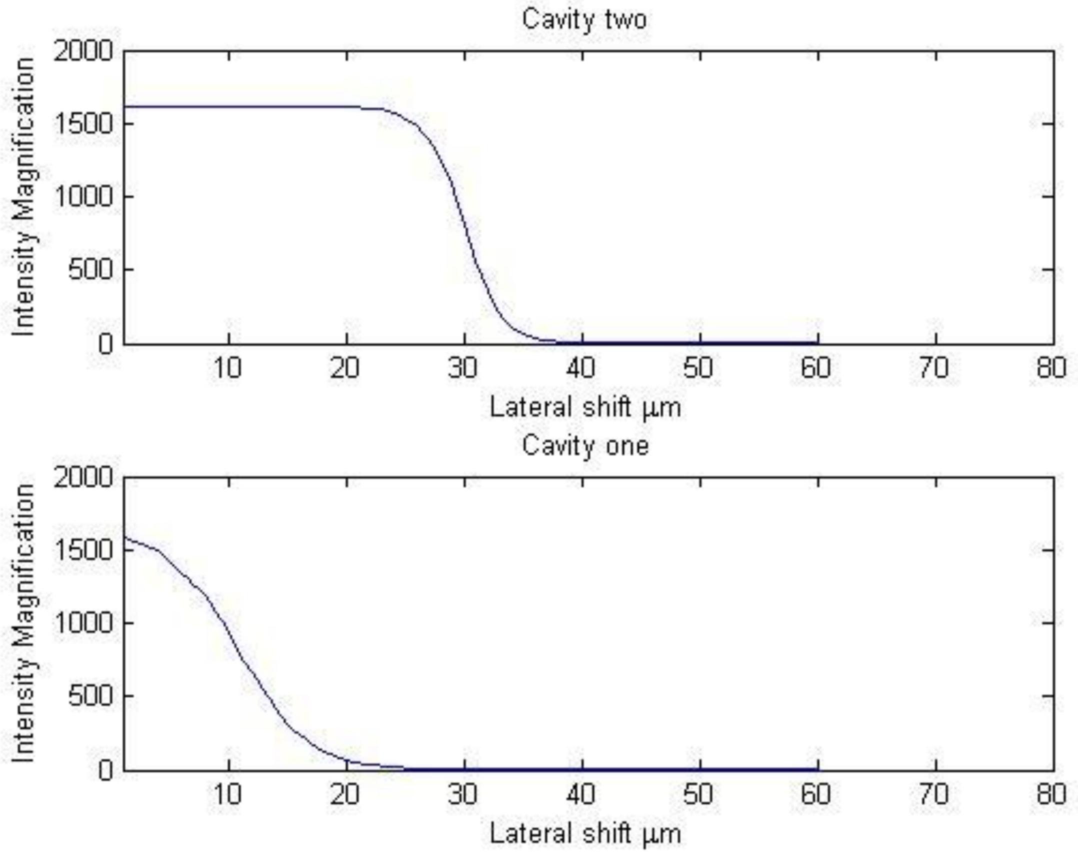
Cavity Variables

Variables	Cavity One	Cavity Two
Radius 1 (r_1)	10 cm	4 cm
Radius 2 (r_2)	∞	∞
Length of Cavity (L)	9.999 cm	3.999 cm
Reflectance 1 (ρ_1)	0.9997	0.9997
Reflectance 2 (ρ_2)	0.9997	0.9997
Wavelength (λ)	852 nm	852 nm
Input Field (E_0)	Gaussian	Gaussian

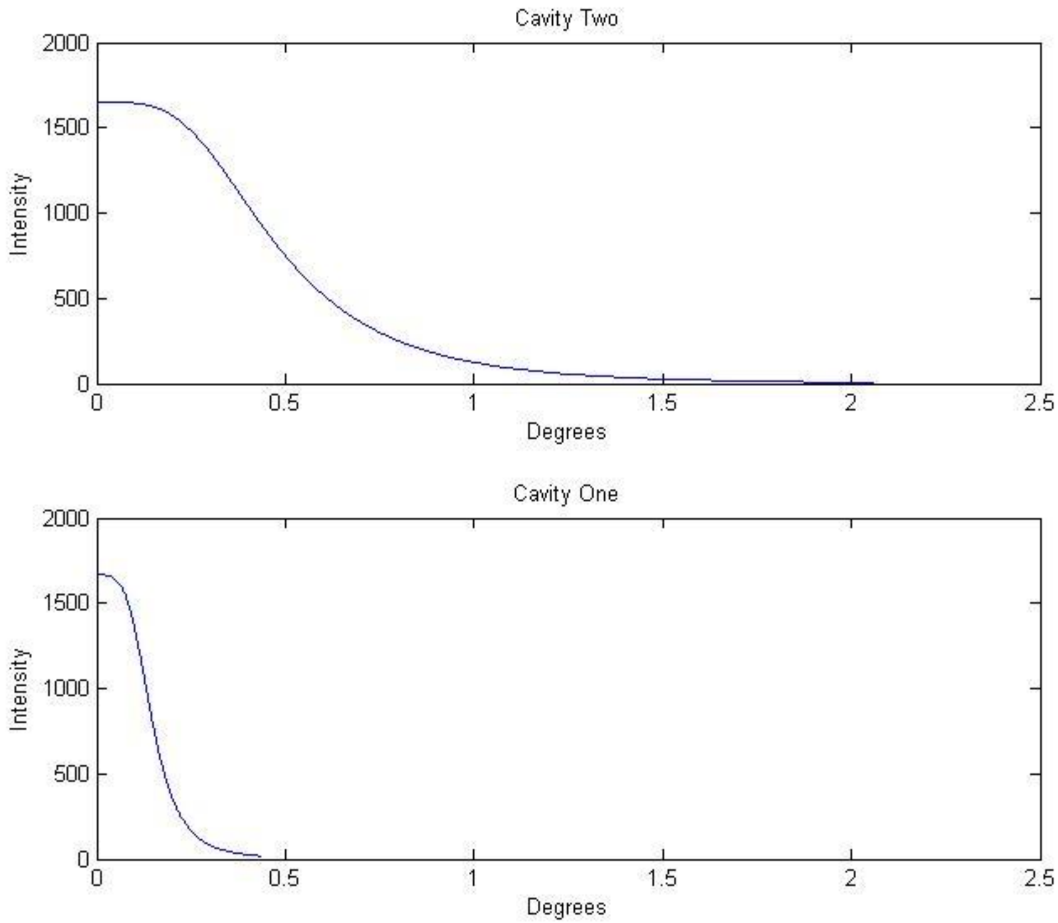
Simulating mirror diameter



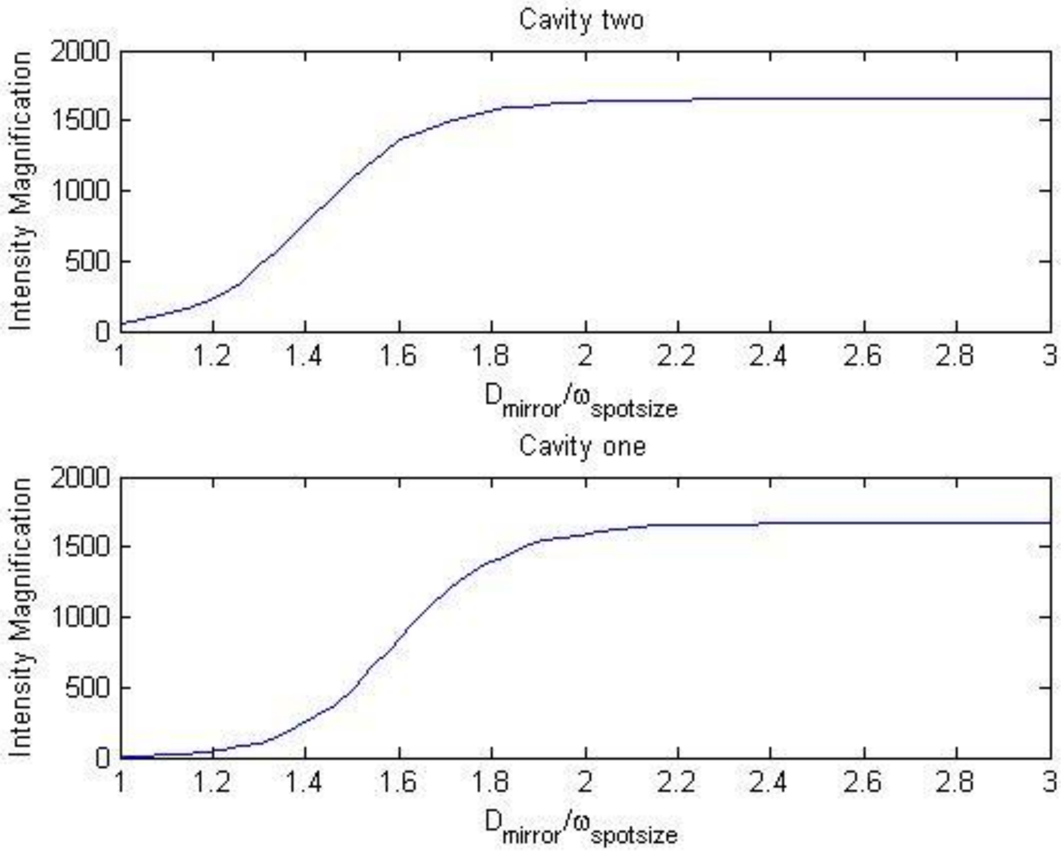
Simulating mirror shift



Simulating mirror tilt



Normalized results mirror diameter



Conclusion

Conclusion

- These cavities are more robust than initially thought.
- The smaller the cavity, the more robust.
- If the cavity can be fabricated, it could prove an invaluable contribution to the opto-mechanical field.